One of the oldest and most frequently used horizontal load-bearing structures in masonry buildings are brick vaults, at first tunnel (barrel) vault and floor arch (flat vault). The tunnel vaults are the most meaningful structure element used in historical architectural and construction engineering from time before Romanesque till the beginning of the 20th century.

The oldest vault was realized in 3300 B.C.;
Roman era – dome vault of Pantheon (span 43.6 m, 27 B.C.);
cathedral Hagia Sofia – dome vault (span 31 m, 532-538 A.D.);
basilica of Maxentius – cross vault (span 25 m, 3rd century A.D);
Czech – 9th century – rotunda of St. Peter (span 8.2 – 8.5 m)
parts of arches

FIG. 4. Arch
KEY:
1. Keystone 7. Centre
2. Vousoirs 8. Span
3. Impost 9. Springing line
6. Intrados or soffit
St. George's basilica

Hall of the Romanesque Palace

The Library Hall of the Clementinum

Vladislav hall

St. George's basilica
Tank (cistern) Istanbul
– 330 a.d.

Barrel vault
Plan of summation of press load in stone pier of gothic vault support system

Nave structure with gothic cross arch
Failures of vault
Barrel vault supported by wall –

span \(1.25 - 3.5\ m\)
load \(1.5 - 3.0\ kN/m^2\)
thickness 150 mm (all structure or crown), 300 mm (impost)

Modification of foot

- Projection foot
- Let-in foot
- Sheare castellation foot

- Larger thickness of wall
- Smaller thickness of wall
- Kinds of springing stone
Barrel vault - Plan of realisation – remove lagging (formwork) 5 -7 days after finish vault – part of creep in bed joint (better contact between mortar and brick - sheare forces and tension)
Valené klenby do travers

- Valené klenby do travers - píné cihly
- Valené klenby do travers - duté cihly
- Valené klenby do “kolejnice”

Barrel vault supported by steel beams (girder)

Modification of foot

Hollow brick

Barrel vault supported by steel beams (girder) – thickened foot

Technique (method) of application of barrel vault supported by steel girder
Barrel vault supported by arch ring

Segmental barrel ring
(angle of support = 90° - central angle)

Segmental barrel vault (angle of support = 60° central angle)
Barrel brick arch ring

Bond of arch ring

Different vertical rise of vault

Different rise

Bond of brick in foot

r/2

R/2

Arch ring – stiffening elements in barrel vault

patka zazubená

shear castellation foot

Modification of foot – joint barrel vault and arch ring
The locations of window openings, where it was not possible to increase the load of interfenestral small pillars, were made arch relieving belts. Filling the walls under arch relieving belts was carried out after settling arch relieving belt. Arch relieving belt also uses the rails so that the load transmitted to the rail load-bearing walls and steel beams were loaded with only the backing masonry under the belt. Respite belt was used in the case of a difficult start, reducing the load bearing brick walls loaded strips, etc.
Vault tie

d) Example of vault tie

e) Adjusting element

f) příklad zajištění klenby kombinovanými kleštěmi
**Static solution of vault**

Equation of equilibrium of vault

![Diagram of static solution of vault](image1)

Equation of equilibrium for rotation

Equation of equilibrium for displacement

Equation of safety strength

**Conditions for capacity check of vault** (carrying-capacity)

1. Equation of equilibrium for rotation – press centre \( c \) mustn’t get out from section
2. Equation of equilibrium for displacement – angle \( \delta \) between resultant force and normal for bed joint must be smaller than angle of friction for masonry (22.5° - wet mortar, 37° - dry mortar)
3. Equation of safety strength - mustn’t get over strength in press on whole structure

**Distribution of normal stress – influence of eccentricity**

- \( e = 0 \)
- \( e < 1/6t \)
- \( e = 1/6t \)
- \( e > 1/6t \)

Resultant force

core of section

- pressure
- tension
Idealized variation of press line in vault

- core of section
- Resultant force line

Failures of vault – change of span (increase)

- 3 hinge
- 4 hinge
Moment and shearing force – different static plan

Arch with 3 hinge

Arch with 2 hinge

Arch without hinge, constant thickness

Arch without hinge, change of thickness (footing)

Arch without hinge, change of thickness (footing)
deformation of barrel masonry vault

The course and magnitude of deformation brick barrel vault [m], thickness 150 / 300 mm, span 3.5 m, 0.7 m surge, $E_{\text{brick}} = 6000 \text{ MPa}$, $E_{\text{mortar}} = 3000 \text{ MPa}$, (a) continuous load of 1 kN / m', B), force of 1 kN at the crown

rise $i = 0.3$m

rise $= 0.7$m
Isolines principal stresses $\sigma_1$ and $\sigma_2$ brick barrel vaults, thick. 150 / 300 mm, span 3.5 m, 0.7 m surge, $E_{\text{brick}} = 6000$ MPa, $E_{\text{mortar}} = 3000$ MPa, continuous load of 1 kN / m, force of 1 kN at the crown.
Isoline of principal stress $\sigma_1$ and $\sigma_2$ barrel masonry vault vertical and horizontal displacement of right support – 10 mm
Characteristic failures of masonry vault

Main cause of failures:

Bad quality of masonry pieces (brick, stone,…) and mortar
Bad depth a quality of bed joints
Bad cohesion and workability of mortar
Bad brick bond
Wrong geometry and section of vault (load, footing)
Wrong plan of load (concentrated load, unsymmetric load, dynamic load,…)
Wrong stiffness and stability of support (vertical, horizontal displacement,…)
Wrong intervention into masonry of support and vault (hole, change of load support, …)

Survey of vault – finding hinge (crush of surface layer and creation of tensile cracks)

Dangerous bed joint – section with extrem stress tension – head of vault = extrados surface (open joint by soffit), center angle more than 120° = soffit surface (open joint by extrados; solution of safety = backing to 1/3 rise vault)
Fixed structure – original plan

Plastic hinge

Change of static plan of vault
maintenance of masonry vault:

a) Vault with disturbed or non bearing filler of bed joint, unblocking masonry elements, unblocking vault ties, ...:
removal of plaster and failure parts of masonry, wedgeing of joint, to depth 30 – 50 mm filling or injection joint by mortar
Injection of local cracks, change or activation ties
Injection of local cracks, maintenance of support structure, foundation
Facing soffit area disturbed by group of cracks by reinforced plaster

b) Vault with large disturbed masonry, large area with cracks, vault with insufficient load capacity may by maintenance by:
Strengthening by arch ring placed on soffit or extrados with distance 2 – 3 m
Suspended vault on steel or reinforced concrete beams or grid
Make extrados vault - vault with insufficient load capacity or large disturbed vault
Make soffit vault - vault with insufficient load capacity or large disturbed vault
Make reinforce concrete slab (plate), suspended disturbed and local maintenance vault

Rehabilitation of vault hanging on reinforced concrete board, beams, grids
extrados reinforced concrete strengthening vault (shell) (thickness 60 – 80 mm)

Reinforced footing rib

Steel net Ø 6-10 mm, 150x150 mm

Separation layer between masonry and RC shell – reduce influence of shrinkage and creep RC

intrados reinforced concrete strengthening vault (shell) (thickness 40 – 80 mm)

Reinforced footing rib

Steel net Ø 6-10 mm

Realization of rib – step by step – length cca 0.8 – 1.2 m
Make hole in masonry vault  Isoline of principal stress $\sigma_1$ and $\sigma_2$
Experimetal works

[Diagram of experimental works with measurements and annotations]
Buckling of right part
Shape of deformation – dependence on load

Vault k10 (375 mm)

Vault k09a (750 mm)

Vault k08a (1000 mm)

Legenda

- 18 kN
- 36 kN
- 72 kN
- 108 kN
- 144 kN
- 180 kN
- max kN

Max = 132 kN

Max = 254 kN

Max = 214 kN
Shape of deformation – dependence on load

a) symmetrical load

vault K3 (rise 375 mm)

Limit force = 68.5 kN

vault K2A (rise 750 mm)

Limit force = 73.1 kN

vault K4 (rise 1000 mm)

Limit force = 80.1 kN

b) non-symmetrical load

vault K5 (rise 375 mm)

Limit force = 12.91 kN

vault K6 (rise 750 mm)

Limit force = 5.61 kN
Without strengthening

a) Nezesílené klenby půdorys

Strengthening by CFRP

b) Zesílení uhlíkovou tkaninou půdorys

Strengthening by HELIFIX (heli bar)

c) Zesílení helikální vyztuží

Strengthening by RC shell

c) Zesílení železobetonovou skořepinou
Limit vertical defomation (mm)

Rise 375 mm

A - without
B - CFRP
C - Heli - intrados
D - Heli - intrados/extrados
E - RC Shell/ Heli - intrados

Rise 750 mm

Klenba

Rise 1000 mm
Flat vault

klenutí pásové

750-1250

20-30

klenutí obyčejné

750-1250

klenutí na střídavý ramenát

nakloněné klenutí - zvýšení prostorové tuhosti klenby

uspořádání stropní konstrukce

úprava ocelových nosníků v průsvitu
Flat vault

Klein’s straight (flat) vault

Iron tape (strip)

Iron tape (strip)

VÝPOČET KLEINOVY DESKY
Concrete vaults supporting by steel profile

NESPALNÉ STROPY Z PROSTÉHO BETONU - BETONOVÉ KLENBY DO TRAVERS

úprava u krajního pole

ocelové táhlo (a´ 3,5m)

ZÁKLADNÍ STATICKÉ ŘEŠENÍ KLENBY

vodorovná síla

H = \frac{qP}{8v}

tloušťka klenby ve vrcholu

t = \frac{H}{100R_{sz}}

tloušťka klenby v patě

T = \frac{H}{100R_{sz} \cos \phi}

tangentu úhlu, který svirá výslednice v patě s vodorovnou přímkou

\text{tg} \phi = \frac{Q}{2H}
Reinforcement concrete roof structure

Monier roof struc.

Monier vault
Masonry Structure

horizontal load bearing structure - vault

Vault

a.

b.

c.

d.

KEY:

a. Tunnel vault
b. Groin vault
c. Rib vault
d. Fan vault

1. Transverse rib
2. Diagonal rib
3. Transverse ridge-rib
4. Longitudinal ridge-rib
5. Tiercerons
6. Liernes
7. Boss
Mason ring of arch ring, rib and vault

Cross vault

Axonometri of romance structure of nave with wood beam roof (St. George’s Basilica)

c) Křížová klenba s čelními valenými pásy

Cross vault with front arch ring

d) Axonometrie románské chrámové lodi s trámovým stropem (sv. Jiří, Pražský hrad)
Vault using in Renaissance  Klenby používané v renesanci

Spherical vault supported by circuit

Spherical vault with small drum (tambour) supported by pendentive

Spherical vault supported by circuit

Klenba valená s lunetou

Barrel vault with welsh groining

d) Dvouvrstvá konstrukce klenbové kopule uložená na osmibokém tamburu

Vault's rib of stone cloister vault

f) Kazetová valená klenba

e) Klenbové žebrí kamenné klášterní klenby
Vault using in Baroque

Segmental sail vault

- Česká placka (plině vysoko kulová, elipsoidická, paraboloidická)
- Česká klenba (nad čtvercovým nebo obdélníkovým půdorysem, česká klenba - rotační elipsoid, nad nepravidelným pětiběžníkem, polokulová stoupající)
- Sail (Bohemian) vault

Double curvature barrel vault (flat vaulted ceiling)

Example of using segmental sail vault

Scheme of vaulting (ground plan)